

1 Asymptotics Potpourri

Algorithm	Best-case	Worst-case	Stable
Selection Sort			
Insertion Sort			
Merge Sort			
Quicksort			
Heapsort			

2 Vertigo

- 2.1 We have a list of N elements that should be sorted, but to our surprise we recently discovered that there are at most k pairs out of order, or k **inversions**, in the list. As a small example, the list $\{ 0, 1, 2, 6, 4, 5, 3 \}$ contains 5 inversions: $(6,4), (6,5), (6,3), (4,3), (5,3)$.

For each value of k below, state the most efficient sorting algorithm and give a tight asymptotic runtime bound.

(a) $k \in O(\log N)$

(b) $k \in O(N)$

(c) $k \in O(N^2)$

3 QuickCo

Malicious Mallory, a sinister hacker, has been hired by a competitor to break into QuickCo, the world leader in sorting algorithms, and tamper with its famous Quicksort algorithms by making them as slow as possible. Mallory succeeded in unlocking the mainframe, but now she needs your help in slowing QuickCo's algorithms down to a halt!

```
int[] data = { 1, 2, 3, 4, 5, 6, 7, 8, 9 };
```

- 3.1 First, Mallory decides to change the way QuickCo chooses a pivot for Quicksort.

Given the `int[] data`, what choice of pivot would cause the worst-case runtime for Quicksort?

- 3.2 Mallory finds an algorithm which always selects the middle element but she is unable to gain write access to it. However, she discovers a way to intercept the incoming data and rearrange it before the algorithm runs.

Given the `int[] data`, rearrange the numbers such that the algorithm will run in its worst-case time.

- 3.3 Does the worst-case runtime of Quicksort depend on the array order, pivot choice, or both? Why?

4 Pivotal Choice

4.1 For each pivot selection strategy below, what is the best, average and worst case runtime?

(a) Always choose the first value in the list.

(b) Always find and choose the median value in the list. Assume finding the median takes $O(N)$ time where N is the length of the list.

(c) Always choose a random pivot.

5 Showdown *Extra for Experts*

5.1 (a) What are the advantages and disadvantages of quicksort?

(b) What are the advantages and disadvantages of merge sort?