1 Fun with Hash Functions (CS 61BL Summer 2014 Midterm 2)

Here are three potential implementations of the Integer’s hashCode() function. Categorize each as either a valid or an invalid hash function. If it is invalid, explain why. If it is valid, point out a flaw/disadvantage.

Note: A "valid" hashCode() means that: any two Integers that are .equals() to each other should also return the same hash code value.

Another note: the Integer class extends the Number class, a direct subclass of Object. The Number class’ hashCode() method directly calls the Object class’ hashCode() method.

(a) public int hashCode() { return -1; }

Valid. As required, this hash function returns the same hashCode for Integers that are .equals() to each other. However, this is a terrible hash code because collisions are extremely frequent (collisions occur 100% of the time).

(b) public int hashCode() { return intValue() * intValue(); }

Valid. Similar to (a), this hash function returns the same hashCode for Integers that are .equals(). However, Integers that share the same absolute values will collide (for example, x=5 and x=-5 will have the same hash code). A better hash function would be to just return the intValue() itself.

(c) public int hashCode() { return super.hashCode(); }

Invalid. This is not a valid hash function because Integers that are .equals() to each other will not have the same hash code. Instead, this hash function returns some integer corresponding to the Integer object’s location in memory.

2 HashMap Modification (CS 61BL Summer 2010 Midterm 2)

(a) When you modify a key that has been inserted into a HashMap will you be able to retrieve that entry again? Explain.

☐ Always □ Sometimes □ Never
If the hashCode for the key happens to change as a result of the modification, then we won’t be able to reliably retrieve the key.

(b) When you modify a value that has been inserted into a HashMap will you be able to retrieve that entry again? Explain.

☐ Always □ Sometimes □ Never
The bucket index for an entry in a HashMap is decided by the key, not the value. Mutating the value does not affect the lookup procedure.
3 Analyzing Project 1’s ArrayDeque Runtime

Recall the ArrayDeque from proj1. Our implementation uses a circular array with two pointers denoting the front and back of the ArrayDeque. Starting with an initial size of 8, the array doubles in size when it reaches full capacity, and halves in size when it’s load factor is lower than 0.25. Resizing will reposition the elements to start from index 0 for ease of maintenance. Fill in this table with best, worst, and average case runtimes of the ArrayDeque methods in Θ(·) notation.

<table>
<thead>
<tr>
<th>Method</th>
<th>Best Case</th>
<th>Worst Case</th>
<th>Average Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>addFirst/Last</td>
<td>Θ(1)</td>
<td>Θ(N)</td>
<td>Θ(1)</td>
</tr>
<tr>
<td>rmFirst/Last</td>
<td>Θ(1)</td>
<td>Θ(N)</td>
<td>Θ(1)</td>
</tr>
<tr>
<td>get</td>
<td>Θ(1)</td>
<td>Θ(1)</td>
<td>Θ(1)</td>
</tr>
</tbody>
</table>

4 Recursion and Dynamic Programming

Implement Fibonacci using memoization (memorizing previously solved problems).

```java
public class Fibonacci {
    HashMap<Integer, Integer> map = new HashMap<Integer, Integer>();

    // fib(0) = 0, fib(1) = 1, fib(2) = 1, fib(3) = 2, ...
    public int fib(int n) {
        if (n == 0) {
            return 0;
        } else if (n == 1) {
            return 1;
        } else if (map.containsKey(n)) {
            return map.get(n);
        }

        int result = fib(n - 2) + fib(n - 1);
        map.put(n, result);
        return result;
    }
}
```
5 Hashing a Tic-Tac-Toe Board (Bonus)

Given the provided (minimal) implementations below, write the `.hashCode()` and `.equals()` methods for the Piece and Board classes (we did Piece.equals() for you). Try to ensure that different board configurations have different hash codes.

```java
public class Piece {
    private String type; // Will be either "X" or "O".

    public boolean equals(Object o) {
        Piece otherPiece = (Piece) o;
        return this.type.equals(otherPiece.type);
    }

    public int hashCode() {
        if (type.equals("X")) {
            return 2;
        } else if (type.equals("O")) {
            return 1;
        }
        return 0;
    }
}

A Piece is either "X" or "O" and can be uniquely represented in two states. The value 0 represents an empty piece (see Board::hashCode) but should never be returned from Piece::hashCode.

```java
public class Board {
    public static final int SIZE = 3; // Tic-Tac-Toe Boards are always 3x3

    private Piece[][] pieces;
    private boolean isXTurn;

    public int hashCode() {
        int code = 0;
        if (isXTurn) {
            code = 1;
        }
        for (int i = 0; i < SIZE; i++) {
            for (int j = 0; j < SIZE; j++) {
                Piece currentPiece = pieces[i][j];
                code *= 3;
                if (currentPiece != null) {
                    code += currentPiece.hashCode();
                }
            }
        }
        return code;
    }
}
```

We multiply by 3 to uniquely identify the particular position for an element, the same way we multiply by 10 to differentiate between 61 and 610.
Intuitively, we can think of Board::hashCode as taking a numeric representation of the board like "XX OOXO O" and turning it into a number like 220112101.

**Bite-sized Bonus:** How do you implement the .equals() method for Board?

Take a **double for** loop through the pieces array and ensure that at every spot, either they are both nulls, both X’s, or both O’s. We didn’t leave space in the above code because we tried to keep everything on one page...

**Bigger Bonus:** Is it possible to perform a "perfect hash"? If we now wanted to have three different types of pieces, X’s, O’s and Triangles, does that change your answer?

Yes, there are a combined $2 \times 3^9$ possible configurations, which can be uniquely represented using the hash code. To include a new type of piece, use powers of 4 to represent the possible pieces in each board location.