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## 1 Fun with Hash Functions (CS 61BL Summer 2014 Midterm 2)

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Here are three potential implementations of the `Integer`'s `hashCode()` function. Categorize each as either a valid or an invalid hash function. If it is invalid, explain why. If it is valid, point out a flaw/disadvantage.

Note: A "valid" `hashCode()` means that: any two `Integers` that are `.equals()` to each other should also return the same hash code value.

Another note: the `Integer` class extends the `Number` class, a direct subclass of `Object`. The `Number` class' `hashCode()` method directly calls the `Object` class' `hashCode()` method.

(a) `public int hashCode() {return -1; }`

Valid. As required, **this** hash function returns the same hash code **for** `Integers` that are `.equals()` to each other. However, **this** is a terrible hash code because collisions are extremely frequent (collisions occur 100% of the time).

(b) `public int hashCode() {return intValue() * intValue(); }`

Valid. Similar to (a), **this** hash function returns the same hash code **for** `Integers` that are `.equals()`. However, `Integers` that share the same absolute values will collide (**for** example, `x=5` and `x=-5` will have the same hash code). A better hash function would be to just **return** the `intValue()` itself.

(c) `public int hashCode() {return super.hashCode(); }`

Invalid. This is not a valid hash function because `Integers` that are `.equals()` to each other will not have the same hash code. Instead, **this** hash function returns some integer corresponding to the `Integer` object's location in memory.

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## 2 HashMap Modification (CS 61BL Summer 2010 Midterm 2)

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(a) When you modify a key that has been inserted into a `HashMap` will you be able to retrieve that entry again? Explain.

Always       Sometimes       Never

If the hash code **for** the key happens to change as a result of the modification, then we won't be able to reliably retrieve the key.

(b) When you modify a value that has been inserted into a `HashMap` will you be able to retrieve that entry again? Explain.

Always       Sometimes       Never

The bucket index **for** an entry in a `HashMap` is decided by the key, not the value. Mutating the value does not affect the lookup procedure.

### 3 Analyzing Project 1's ArrayDeque Runtime

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Recall the `ArrayDeque` from proj1. Our implementation uses a circular array with two pointers denoting the front and back of the `ArrayDeque`. Starting with an initial size of 8, the array doubles in size when it reaches full capacity, and halves in size when it's load factor is lower than 0.25. Resizing will reposition the elements to start from index 0 for ease of maintenance. Fill in this table with best, worst, and average case runtimes of the `ArrayDeque` methods in  $\Theta(\cdot)$  notation.

	Best Case	Worst Case	Average Case
<code>addFirst/Last</code>	$\Theta(1)$	$\Theta(N)$	$\Theta(1)$
<code>rmFirst/Last</code>	$\Theta(1)$	$\Theta(N)$	$\Theta(1)$
<code>get</code>	$\Theta(1)$	$\Theta(1)$	$\Theta(1)$

### 4 Recursion and Dynamic Programming

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Implement Fibonacci using memoization (memorizing previously solved problems).

```
public class Fibonacci {
    HashMap<Integer, Integer> map = new HashMap<Integer, Integer>();

    // fib(0) = 0, fib(1) = 1, fib(2) = 1, fib(3) = 2, ...
    public int fib(int n) {
        if (n == 0) {
            return 0;
        } else if (n == 1) {
            return 1;
        } else if (map.containsKey(n)) {
            return map.get(n);
        }

        int result = fib(n - 2) + fib(n - 1);
        map.put(n, result);
        return result;
    }
}
```

## 5 Hashing a Tic-Tac-Toe Board (Bonus)

Given the provided (minimal) implementations below, write the `.hashCode()` and `.equals()` methods for the `Piece` and `Board` classes (we did `Piece.equals()` for you). Try to ensure that different board configurations have different hash codes.

```
public class Piece {
    private String type; // Will be either "X" or "O".

    public boolean equals(Object o) {
        Piece otherPiece = (Piece) o;
        return this.type.equals(otherPiece.type);
    }

    public int hashCode() {
        if (type.equals("X")) {
            return 2;
        } else if (type.equals("O")) {
            return 1;
        }
        return 0;
    }
}
```

A `Piece` is either "X" or "O" and can be uniquely represented in two states. The value 0 represents an empty piece (see `Board::hashCode`) but should never be returned from `Piece::hashCode`.

```
public class Board {
    public static final int SIZE = 3; // Tic-Tac-Toe Boards are always 3x3

    private Piece[][] pieces;
    private boolean isXTurn;

    public int hashCode() {
        int code = 0;
        if (isXTurn) {
            code = 1;
        }

        for (int i = 0; i < SIZE; i++) {
            for (int j = 0; j < SIZE; j++) {
                Piece currentPiece = pieces[i][j];
                code *= 3;
                if (currentPiece != null) {
                    code += currentPiece.hashCode();
                }
            }
        }

        return code;
    }
}
```

We multiply by 3 to uniquely identify the particular position **for** an element, the same way we multiply by 10 to differentiate between 61 and 610.

Intuitively, we can think of `Board::hashCode` as taking a numeric representation of the board like "XX OOXO O" and turning it into a number like 220112101.

**Bite-sized Bonus:** How do you implement the `.equals()` method for `Board`?

Take a [`double`] `for` loop through the `pieces` array and ensure that at every spot, either they are both nulls, both X's, or both O's. We didn't leave space in the above code because we tried to keep everything on one page...

**Bigger Bonus:** Is it possible to perform a "perfect hash"? If we now wanted to have three different types of pieces, X's, O's and Triangles, does that change your answer?

Yes, there are a combined  $2 \times 3^9$  possible configurations, which can be uniquely represented using the hash code. To include a `new` type of piece, use powers of 4 to represent the possible pieces in each board location.