1 Welcome O & Omega, Θ's Not Alone!

Order the following big-O runtimes from most to least efficient:

\[ O(\log n), O(1), O(n^2), O(n^3), O(n \log n), O(n), O(n!), O(2^n), O(n^2 \log n) \]

\[ O(1) \subset O(\log n) \subset O(n) \subset O(n \log n) \subset O(n^2) \subset O(n^3) \subset O(2^n) \subset O(n!) \subset O(n^2) \]

Are the statements in the right column true or false? If false, correct the asymptotic notation (Ω, Θ, O). Be sure to give the tightest bound. Ω(·) is the opposite of O(·), i.e. \( f(n) = Ω(g(n)) \iff g(n) = O(f(n)) \).

\[
\begin{align*}
    f(n) = 20501 & \quad g(n) = 1 & \quad f(n) \in O(g(n)) & \quad True^* \\
    f(n) = n^2 + n & \quad g(n) = 0.000001 n^3 & \quad f(n) \in Ω(g(n)) & \quad False^{**} O \\
    f(n) = 2^n + 1000 & \quad g(n) = 4^n + n^{100} & \quad f(n) \in O(g(n)) & \quad True \\
    f(n) = \log(n)^{100} & \quad g(n) = n \log n & \quad f(n) \in Θ(g(n)) & \quad True \\
    f(n) = n \log n + 3^n + n & \quad g(n) = n^2 + n + \log n & \quad f(n) \in Ω(g(n)) & \quad True \\
    f(n) = n \log n + n^2 & \quad g(n) = \log n + n^2 & \quad f(n) \in Θ(g(n)) & \quad True \\
    f(n) = n \log n & \quad g(n) = (\log n)^2 & \quad f(n) \in O(g(n)) & \quad False \Omega \\
\end{align*}
\]

*The 1st and 3rd ones are true, but Θ is a better bound.  
**Even though \( n^3 \) is strictly worse than \( n^2 \), \( n^2 \) is still in \( O(n^3) \) because \( n^2 \) is always as good as or better than \( n^3 \) and can never be worse.

2 Analyzing Runtime

Give the worst case runtime in \( Ω(·) \) notation. Extra: Give the best case runtime in \( Ω(·) \).

A. Use \( M \) and \( N \) in your result. \texttt{ping()} is a constant time, \( O(1) \), function that returns an int.

```
int j = 0,
for (int i = N; i > 0; i--) {
    for (; j <= M; j++) // Can do without {} because there is "1" line after
        if (ping(i, j) > 64)
            break; // ends the loop (only 1 loop, so only the inner loop here)
}
```

B. Use \( N \) in your result, where \( N \) is the length of \texttt{arr}.

```
public static boolean mystery(int[] arr) {
    arr = mrpoolsort(arr); // creates sorted copy of arr in \( Θ(N \log N) \) time
    int N = arr.length;
    for (int i = 0; i < N; i += 1) {
        boolean x = false;
        for (int j = 0; j < N; j += 1) {
        }
    }
}
```
Achilles Added Additional Amazing Asymptotic And Algorithmic Analysis Achievements:

What is mystery() doing?

mystery() returns true if every int has a duplicate in the array (1,2,1,2 would return true) and false if there is any unique int in the array (1,2,2 is false).

Using an ADT, can you rewrite mystery() with a better runtime? What about if we make the assumption an int can appear in arr at most twice, is there a better way (uses constant memory)?

A $\Theta(N)$ algorithm is to use a Map and do key=element and value=number of appearances, then make sure all values are >1. Uses $O(N)$ memory however.

There is none - XOR does not work for 1, 2, 3. Can do constant space by sorting then going through, but it's $O(n\log n)$

3 Have You Ever Went Fast?

Given an integer x and a sorted array A[] of N distinct integers, design an algorithm to find if there exists indices i and j such that A[i] + A[j] == x.

Let’s start with the naive solution:

```java
public static boolean findSum(int[] A, int x) {
    for (int i = 0; i < A.length; i++)
        for (int j = 0; j < A.length; j++)
                return true;
    return false;
}
```

Can we do this faster? Hint: Does order matter here?

```java
public static boolean findSumFaster(int[] A, int x) {
    int left = 0;
    int right = A.length - 1;
    while (left <= right){
            return true;
            left++;
        else
            right--;
    }
    return false;
}
```
What is the runtime of both these algorithms?
Naive: Worst = $O(N^2)$, Best = $\Omega(1)$.
Optimized: Worst = $O(N)$, Best = $\Omega(1)$

4 Basic Interview Type Questions (Extra for Experts)

**Union:** Write the code that returns an array that is the union between two given arrays. The union of two arrays is a list that includes everything that is in both arrays, with no duplicates. Assume the given arrays do not contain duplicates. Ex: Union of 1,2,3,4 and 3,4,5,6 is 1,2,3,4,5,6
Hint: The method should run in $O(M + N)$ time where $M$ and $N$ are the sizes of the two arrays.

```java
public static int[] union(int[] A, int[] B) {
    HashSet<Integer> set = new HashSet<Integer>();
    for (int num : A) {
        set.add(num);
    }
    for (int num : B) {
        set.add(num);
    }
    int[] unionArray = new int[set.size()];
    int index = 0;
    for (int num : set) {
        unionArray[index] = num;
        index += 1;
    }
    return unionArray;
}
```

**Intersection:** Now do the same as above, but find the intersection between both arrays. The intersection of two arrays is the list of all elements that are in both arrays. Again assume that neither array has duplicates. Ex: Intersection of 1,2,3,4 and 3,4,5,6 is 3,4.
Hint: Think about using ADTs other than arrays to make the code more efficient.

```java
public static int[] intersection(int[] A, int[] B) {
    HashSet<Integer> setOfA = new HashSet<Integer>();
    HashSet<Integer> intersectionSet = new HashSet<Integer>();
    for (int num : A) {
        setOfA.add(num);
    }
    for (int num : B) {
        if (setOfA.contains(num)) {
            intersectionSet.add(num);
        }
    }
    int[] intersectionArray = new int[intersectionSet.size()];
    int index = 0;
    for (int num : intersectionSet) {
        intersectionArray[index] = num;
        index += 1;
    }
    return intersectionArray;
}
```
What is the runtime, \( \Omega(\cdot) \) and \( O(\cdot) \), of the above algorithm?

\( \Theta(N + M) \)