CS 61B Discussion 12 Spring 2015

1 Quicksort

(a) Sort the following unordered list using in-place quicksort. Assume that the pivot you use is always the first element. Show the steps taken at each partitioning step.

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34, 25, 82, 34, 28, 16, 75, 96
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(b) What is the worst case running time of quicksort? Give an example of a list that meets this worst case running time.

(c) What is the best case running time of quicksort? Briefly justify why you can't do any better than this best case running time.

(d) What are two techniques that can be used to reduce the probability of quicksort taking the worst case running time?

(e) What are the best and worst case running times of quickselect?

2 Comparing Sorting Algorithms

When choosing an appropriate algorithm, there are often several tradeoffs that we have to consider. For the following sorting algorithms, give the expected space complexity, time complexity, and whether or not each sort is stable.

	Time Complexity	Space Complexity	Stable?
Insertion Sort			
Heapsort			
Mergesort			
Quicksort			

(a) For each unstable sort, give an example of a list where the order of equivalent items is not preserved.

(b) In general, what are some other tradeoffs we might want to consider when designing an algorithm?

3 Bounding Practice

Given an array, the heapification operation permutes the elements of the array into a heap. There are many solutions to the heapification problem. One approach is bottom-up heapification, which calls `sink(x)` on all nodes in reverse level order. Another is top-down heapification, which calls `swim(x)` on all nodes in level order.

(a) Why can we say that any solution for heapification requires $\Omega(n)$ time?

(b) Give the worst-case runtime for top-down heapification in $\Theta(\cdot)$ notation. Why does this mean that the optimal solution for heapification takes $O(n \log n)$ time?

(c) **Matthew's Mathematical Magic for Moguls:** Show that the running time of bottom-up heapify is $\Theta(n)$. Is bottom-up heapification asymptotically optimal?